

$$17. \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array} \right] \quad 18. \left[\begin{array}{cccc|c} 1 & -2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array} \right]$$

Write the linear system corresponding to each reduced augmented matrix in Problems 19–28 and write the solution of the system.

$$19. \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$20. \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right]$$

$$21. \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$22. \left[\begin{array}{ccc|c} 1 & -2 & 0 & -3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$23. \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$24. \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array} \right]$$

$$25. \left[\begin{array}{ccc|c} 1 & 0 & -3 & 5 \\ 0 & 1 & 2 & -7 \end{array} \right]$$

$$26. \left[\begin{array}{ccc|c} 1 & 0 & 1 & -4 \\ 0 & 1 & -1 & 6 \end{array} \right]$$

$$27. \left[\begin{array}{cccc|c} 1 & -2 & 0 & -3 & -5 \\ 0 & 0 & 1 & 3 & 2 \end{array} \right]$$

$$28. \left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & 4 \\ 0 & 1 & -1 & 2 & -1 \end{array} \right]$$

29. In which of Problems 19, 21, 23, 25, and 27 is the number of leftmost ones equal to the number of variables?

30. In which of Problems 20, 22, 24, 26, and 28 is the number of leftmost ones equal to the number of variables?

31. In which of Problems 19, 21, 23, 25, and 27 is the number of leftmost ones less than the number of variables?

32. In which of Problems 20, 22, 24, 26, and 28 is the number of leftmost ones less than the number of variables?

 In Problems 33–38, discuss the validity of each statement about linear systems. If the statement is always true, explain why. If not, give a counterexample.

33. If the number of leftmost ones is equal to the number of variables, then the system has exactly one solution.

34. If the number of leftmost ones is less than the number of variables, then the system has infinitely many solutions.

35. If the number of leftmost ones is equal to the number of variables and the system is consistent, then the system has exactly one solution.

36. If the number of leftmost ones is less than the number of variables and the system is consistent, then the system has infinitely many solutions.

37. The number of equations is less than or equal to the number of variables.

38. The number of leftmost ones is less than or equal to the number of equations.

Use row operations to change each matrix in Problems 39–46 to reduced form.

$$39. \left[\begin{array}{cc|c} 1 & 2 & -1 \\ 0 & 1 & 3 \end{array} \right]$$

$$40. \left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & 2 & -4 \end{array} \right]$$

$$41. \left[\begin{array}{ccc|c} 1 & 1 & 1 & 16 \\ 2 & 3 & 4 & 25 \end{array} \right]$$

$$42. \left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 3 & 5 & 7 & 30 \end{array} \right]$$

$$43. \left[\begin{array}{ccc|c} 1 & 0 & -3 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 3 & -6 \end{array} \right]$$

$$44. \left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & -2 & 2 \end{array} \right]$$

$$45. \left[\begin{array}{ccc|c} 1 & 2 & -2 & -1 \\ 0 & 3 & -6 & 1 \\ 0 & -1 & 2 & -\frac{1}{3} \end{array} \right]$$

$$46. \left[\begin{array}{ccc|c} 0 & -2 & 8 & 1 \\ 2 & -2 & 6 & -4 \\ 0 & -1 & 4 & \frac{1}{2} \end{array} \right]$$

Solve Problems 47–62 using Gauss–Jordan elimination.

$$47. \begin{aligned} 2x_1 + 4x_2 - 10x_3 &= -2 \\ 3x_1 + 9x_2 - 21x_3 &= 0 \\ x_1 + 5x_2 - 12x_3 &= 1 \end{aligned}$$

$$48. \begin{aligned} 3x_1 + 5x_2 - x_3 &= -7 \\ x_1 + x_2 + x_3 &= -1 \\ 2x_1 + 11x_3 &= 7 \end{aligned}$$

$$49. \begin{aligned} 3x_1 + 8x_2 - x_3 &= -18 \\ 2x_1 + x_2 + 5x_3 &= 8 \\ 2x_1 + 4x_2 + 2x_3 &= -4 \end{aligned}$$

$$50. \begin{aligned} 2x_1 + 6x_2 + 15x_3 &= -12 \\ 4x_1 + 7x_2 + 13x_3 &= -10 \\ 3x_1 + 6x_2 + 12x_3 &= -9 \end{aligned}$$

$$51. \begin{aligned} 2x_1 - x_2 - 3x_3 &= 8 \\ x_1 - 2x_2 &= 7 \end{aligned}$$

$$52. \begin{aligned} 2x_1 + 4x_2 - 6x_3 &= 10 \\ 3x_1 + 3x_2 - 3x_3 &= 6 \end{aligned}$$

$$53. \begin{aligned} 2x_1 - x_2 &= 0 \\ 3x_1 + 2x_2 &= 7 \\ x_1 - x_2 &= -1 \end{aligned}$$

$$54. \begin{aligned} 2x_1 - x_2 &= 0 \\ 3x_1 + 2x_2 &= 7 \\ x_1 - x_2 &= -2 \end{aligned}$$

$$55. \begin{aligned} 3x_1 - 4x_2 - x_3 &= 1 \\ 2x_1 - 3x_2 + x_3 &= 1 \\ x_1 - 2x_2 + 3x_3 &= 2 \end{aligned}$$

56. $3x_1 + 7x_2 - x_3 = 11$

$x_1 + 2x_2 - x_3 = 3$

$2x_1 + 4x_2 - 2x_3 = 10$

57. $3x_1 - 2x_2 + x_3 = -7$

$2x_1 + x_2 - 4x_3 = 0$

$x_1 + x_2 - 3x_3 = 1$

58. $2x_1 + 3x_2 + 5x_3 = 21$

$x_1 - x_2 - 5x_3 = -2$

$2x_1 + x_2 - x_3 = 11$

59. $2x_1 + 4x_2 - 2x_3 = 2$

$-3x_1 - 6x_2 + 3x_3 = -3$

60. $3x_1 - 9x_2 + 12x_3 = 6$

$-2x_1 + 6x_2 - 8x_3 = -4$

61. $4x_1 - x_2 + 2x_3 = 3$

$-4x_1 + x_2 - 3x_3 = -10$

$8x_1 - 2x_2 + 9x_3 = -1$

62. $4x_1 - 2x_2 + 2x_3 = 5$

$-6x_1 + 3x_2 - 3x_3 = -2$

$10x_1 - 5x_2 + 9x_3 = 4$

-  63. Consider a consistent system of three linear equations in three variables. Discuss the nature of the system and its solution set if the reduced form of the augmented coefficient matrix has

(A) One leftmost 1 (B) Two leftmost 1's

(C) Three leftmost 1's (D) Four leftmost 1's

-  64. Consider a system of three linear equations in three variables. Give examples of two reduced forms that are not row-equivalent if the system is

(A) Consistent and dependent (B) Inconsistent

Solve Problems 65–70 using Gauss–Jordan elimination.

65. $x_1 + 2x_2 - 4x_3 - x_4 = 7$

$2x_1 + 5x_2 - 9x_3 - 4x_4 = 16$

$x_1 + 5x_2 - 7x_3 - 7x_4 = 13$

66. $2x_1 + 4x_2 + 5x_3 + 4x_4 = 8$

$x_1 + 2x_2 + 2x_3 + x_4 = 3$

67. $x_1 - x_2 + 3x_3 - 2x_4 = 1$

$-2x_1 + 4x_2 - 3x_3 + x_4 = 0.5$

$3x_1 - x_2 + 10x_3 - 4x_4 = 2.9$

$4x_1 - 3x_2 + 8x_3 - 2x_4 = 0.6$

68. $x_1 + x_2 + 4x_3 + x_4 = 1.3$

$-x_1 + x_2 - x_3 = 1.1$

$2x_1 + x_3 + 3x_4 = -4.4$

$2x_1 + 5x_2 + 11x_3 + 3x_4 = 5.6$

69. $x_1 - 2x_2 + x_3 + x_4 + 2x_5 = 2$

$-2x_1 + 4x_2 + 2x_3 + 2x_4 - 2x_5 = 0$

$3x_1 - 6x_2 + x_3 + x_4 + 5x_5 = 4$

$-x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 3$

70. $x_1 - 3x_2 + x_3 + x_4 + 2x_5 = 2$

$-x_1 + 5x_2 + 2x_3 + 2x_4 - 2x_5 = 0$

$2x_1 - 6x_2 + 2x_3 + 2x_4 + 4x_5 = 4$

$-x_1 + 3x_2 - x_3 + x_5 = -3$

71. Find a , b , and c so that the graph of the quadratic equation $y = ax^2 + bx + c$ passes through the points $(-2, 9)$, $(1, -9)$, and $(4, 9)$.

72. Find a , b , and c so that the graph of the quadratic equation $y = ax^2 + bx + c$ passes through the points $(-1, -5)$, $(2, 7)$, and $(5, 1)$.

Applications

Construct a mathematical model for each of the following problems. (The answers in the back of the book include both the mathematical model and the interpretation of its solution.) Use Gauss–Jordan elimination to solve the model and then interpret the solution.

-  73. **Boat production.** A small manufacturing plant makes three types of inflatable boats: one-person, two-person, and four-person models. Each boat requires the services of three departments, as listed in the table. The cutting, assembly, and packaging departments have available a maximum of 380, 330, and 120 labor-hours per week, respectively.

	One-Person	Two-Person	Four-Person
Department	Boat	Boat	Boat
Cutting	0.5 hr	1.0 hr	1.5 hr
Assembly	0.6 hr	0.9 hr	1.2 hr
Packaging	0.2 hr	0.3 hr	0.5 hr

- (A) How many boats of each type must be produced each week for the plant to operate at full capacity?
 (B) How is the production schedule in part (A) affected if the packaging department is no longer used?
 (C) How is the production schedule in part (A) affected if the four-person boat is no longer produced?

-  74. **Production scheduling.** Repeat Problem 73 assuming that the cutting, assembly, and packaging departments have available a maximum of 350, 330, and 115 labor-hours per week, respectively.

75. **Tank car leases.** A chemical manufacturer wants to lease a fleet of 24 railroad tank cars with a combined carrying capacity of 520,000 gallons. Tank cars with three different carrying capacities are available: 8,000 gallons, 16,000 gallons, and 24,000 gallons. How many of each type of tank car should be leased?
76. **Airplane leases.** A corporation wants to lease a fleet of 12 airplanes with a combined carrying capacity of 220 passengers. The three available types of planes carry 10, 15, and 20 passengers, respectively. How many of each type of plane should be leased?
77. **Tank car leases.** Refer to Problem 75. The cost of leasing an 8,000-gallon tank car is \$450 per month, a 16,000-gallon tank car is \$650 per month, and a 24,000-gallon tank car is \$1,150 per month. Which of the solutions to Problem 75 would minimize the monthly leasing cost?

In Problems 21–24, find x_1 and x_2 .

$$21. \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \quad 22. \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

$$23. \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 10 \end{bmatrix} \quad 24. \begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

In Problems 25–30, solve for x_1 and x_2 .

$$25. \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \end{bmatrix}$$

$$26. \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

$$27. \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$28. \begin{bmatrix} 3 & -4 \\ -6 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$29. \begin{bmatrix} 3 & -2 \\ 6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$30. \begin{bmatrix} 3 & -1 \\ 6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

In Problems 31–38, write each system as a matrix equation and solve using inverses. [Note: The inverses were found in Problems 41–48, Exercises 4.5.]

$$31. \begin{aligned} x_1 + 2x_2 &= k_1 \\ x_1 + 3x_2 &= k_2 \end{aligned}$$

(A) $k_1 = 1, k_2 = 3$
 (B) $k_1 = 3, k_2 = 5$
 (C) $k_1 = -2, k_2 = 1$

$$32. \begin{aligned} 2x_1 + x_2 &= k_1 \\ 5x_1 + 3x_2 &= k_2 \end{aligned}$$

(A) $k_1 = 2, k_2 = 13$
 (B) $k_1 = 2, k_2 = 4$
 (C) $k_1 = 1, k_2 = -3$

$$33. \begin{aligned} x_1 + 3x_2 &= k_1 \\ 2x_1 + 7x_2 &= k_2 \end{aligned}$$

(A) $k_1 = 2, k_2 = -1$
 (B) $k_1 = 1, k_2 = 0$
 (C) $k_1 = 3, k_2 = -1$

$$34. \begin{aligned} 2x_1 + x_2 &= k_1 \\ x_1 + x_2 &= k_2 \end{aligned}$$

(A) $k_1 = -1, k_2 = -2$
 (B) $k_1 = 2, k_2 = 3$
 (C) $k_1 = 2, k_2 = 0$

$$35. \begin{aligned} x_1 - 3x_2 &= k_1 \\ x_2 + x_3 &= k_2 \\ 2x_1 - x_2 + 4x_3 &= k_3 \end{aligned}$$

(A) $k_1 = 1, k_2 = 0, k_3 = 2$
 (B) $k_1 = -1, k_2 = 1, k_3 = 0$
 (C) $k_1 = 2, k_2 = -2, k_3 = 1$

$$36. \begin{aligned} 2x_1 + 3x_2 &= k_1 \\ x_1 + 2x_2 + 3x_3 &= k_2 \\ -x_2 - 5x_3 &= k_3 \end{aligned}$$

(A) $k_1 = 0, k_2 = 2, k_3 = 1$
 (B) $k_1 = -2, k_2 = 0, k_3 = 1$
 (C) $k_1 = 3, k_2 = 1, k_3 = 0$

$$37. \begin{aligned} x_1 + x_2 &= k_1 \\ 2x_1 + 3x_2 - x_3 &= k_2 \\ x_1 + 2x_3 &= k_3 \end{aligned}$$

(A) $k_1 = 2, k_2 = 0, k_3 = 4$
 (B) $k_1 = 0, k_2 = 4, k_3 = -2$
 (C) $k_1 = 4, k_2 = 2, k_3 = 0$

$$38. \begin{aligned} x_1 - x_3 &= k_1 \\ 2x_1 - x_2 &= k_2 \\ x_1 + x_2 - 2x_3 &= k_3 \end{aligned}$$

(A) $k_1 = 4, k_2 = 8, k_3 = 0$
 (B) $k_1 = 4, k_2 = 0, k_3 = -4$
 (C) $k_1 = 0, k_2 = 8, k_3 = -8$

 In Problems 39–44, the matrix equation is not solved correctly. Explain the mistake and find the correct solution. Assume that the indicated inverses exist.

$$39. AX = B, X = \frac{B}{A}$$

$$40. XA = B, X = \frac{B}{A}$$

$$41. XA = B, X = A^{-1}B$$

$$42. AX = B, X = BA^{-1}$$

$$43. AX = BA, X = A^{-1}BA, X = B$$

$$44. XA = AB, X = AB A^{-1}, X = B$$

 In Problems 45–50, explain why the system cannot be solved by matrix inverse methods. Discuss methods that could be used and then solve the system.

$$45. \begin{aligned} -2x_1 + 4x_2 &= -5 \\ 6x_1 - 12x_2 &= 15 \end{aligned}$$

$$46. \begin{aligned} -2x_1 + 4x_2 &= 5 \\ 6x_1 - 12x_2 &= 15 \end{aligned}$$

$$47. \begin{aligned} x_1 - 3x_2 - 2x_3 &= -1 \\ -2x_1 + 6x_2 + 4x_3 &= 3 \end{aligned}$$

$$48. \begin{aligned} x_1 - 3x_2 - 2x_3 &= -1 \\ -2x_1 + 7x_2 + 3x_3 &= 3 \end{aligned}$$

$$49. \begin{aligned} x_1 - 2x_2 + 3x_3 &= 1 \\ 2x_1 - 3x_2 - 2x_3 &= 3 \\ x_1 - x_2 - 5x_3 &= 2 \end{aligned}$$

$$50. \begin{aligned} x_1 - 2x_2 + 3x_3 &= 1 \\ 2x_1 - 3x_2 - 2x_3 &= 3 \\ x_1 - x_2 - 5x_3 &= 4 \end{aligned}$$